Examples

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EXAMPLE (1)

Suppose you wish to derive a formula for the distance x traveled by a car in a time of t if the car starts from rest and moves with constant acceleration a. Check the validity of this expression $x = \frac{1}{2} a t^2$.

$$LHS = [L]$$

$$RHS = [LT^{-2}] * [T]^2 = [L]$$

The equation is dimensionally correct.

Show that the expression $\upsilon = \upsilon_0 + a\,t$ is dimensionally correct ?

$$LHS = [LT^{-1}]$$

$$RHS = [LT^{-1}] + [LT^{-2}][T] = [LT^{-1}]$$

The equation is dimensionally correct.

Example (3)

Suppose we are told that the acceleration a of a particle moving in a circle of radius r with uniform velocity υ is proportional to some power of r, say r^n, and some power of υ, say υ^m. How can we determine the powers of r and υ ?

$$a = k\, r^n\, \upsilon^m$$

$$LHS = [LT^{-2}]$$

$$[L]\,[T^{-2}] = [L]^n\,[LT^{-1}]^m = [L]^{n+m}\,[T]^{-m}$$

For [T]; $m = 2$

For [L]; $n + m = 1$ $n = -1$

EXAMPLE (4)

The mass m of the largest stone that can be moved by flowing stream, depends upon the stream velocity υ, the density of the fluid flowing ρ, and the acceleration due to gravity g. Show that m varies with the 6th power of the velocity of flow ?

$$m = k\, \upsilon^a\, \rho^b\, g^c$$

$$[M] = [LT^{-1}]^a\, [M L^{-3}]^b\, [LT^{-2}]^c = [M]^b\, [L]^{a-3b+c}\, [T]^{-a-2c}$$

For [M];	$b = 1$	
For [L];	$0 = a - 3b + c$	$a + c = 3$
For [T];	$0 = -a - 2c$	$a = -2c$
So,	$c = -3$	$a = 6$

$$m = k\, \upsilon^6\, \rho\, g^{-3}$$

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EXAMPLE (5)

The period (τ) of a simple pendulum is the time for one complete swing. How does (τ) depends on the mass (m) of the bob, the length (ℓ) of the string and gravitational (g)?

$$\tau = k\, m^a\, \ell^b\, g^c$$

$$[T] = [M]^a\,[L]^b\,[LT^{-2}]^c = [M]^a\,[L]^{b+c}\,[T]^{-2c}$$

For [M]; $a = 0$

For [T]; $1 = -2c$ $c = -½$

For [L]; $0 = b + c$ $b = ½$

$$\tau = k\, \ell^{½}\, g^{-½}$$

$$\tau = k\sqrt{\frac{\ell}{g}}$$

PROB (1)

Show that the following equation is dimensionally correct?

$$X = \upsilon_0 t + \tfrac{1}{2} a t^2$$

$$LHS = [L]$$

$$RHS = [LT^{-1}] * [T] + [LT^{-2}] * [T]^2 = [L]$$

$$RHS = LHS$$ العلاقة صحيحة

PROB (6)

Newton's law of Gravitation motion $F = G M m / r^2$, where r is the distance between the particles. What are the SI units of G ?

$$F = G m_1 m_2 / r^2$$

$$[M L T^{-2}] = [G] [M] [M] / [L^2] = [G] [M^2] [L^{-2}]$$

$$[G] = [L^3] [M^{-1}] [T^{-2}]$$

ALSO; The unit of G is $m^3/Kg.s^2$

1- The mattress of a water bed is 2.00 m long by 2.00 and 30.0 cm deep.

(A) Find the weight of the water in the mattress.

B) Find the pressure exerted by the water on the floor when the bed rests in its normal position. Assume that the entire lower surface of the bed makes contact with the floor.

$$M = \rho V = (1\,000 \text{ kg/m}^3)(1.20 \text{ m}^3) = 1.20 \times 10^3 \text{ kg}$$

and its weight is

$$Mg = (1.20 \times 10^3 \text{ kg})(9.80 \text{ m/s}^2) = 1.18 \times 10^4 \text{ N}$$

B) When the bed is in its normal position, the area in contact with the floor is 4.00 m^2 ,thus,

$$P = \frac{1.18 \times 10^4 \text{ N}}{4.00 \text{ m}^2} = 2.95 \times 10^3 \text{ Pa}$$

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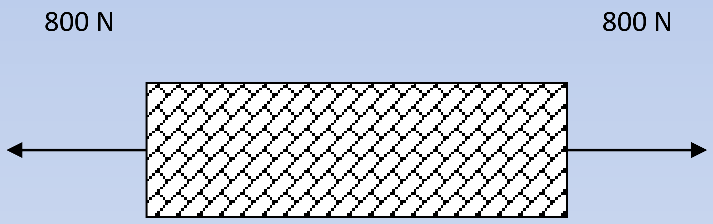
Example

The bar shown has a square cross section for which the depth and thickness are
40 mm. If an axial force of 800 N is applied along the centroidal axis of the bar's cross sectional area, determine the average normal stress acting on the bar ?



$$\sigma = \frac{F}{A} = \frac{800}{(40*10^{-3})^2} = 500*10^3 \text{ Pa}$$

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Example:

A wire of length 120cm and diameter 0.82mm, supported from one end, A 5.3kg in the other end . Find :

- ***The stress in the wire***
- ***The strain in the wire***
- ***The strain energy where $Y = 1.2x\ 10^{12}$ dyne/cm^2 and $g = 980$cm/sec^2***

Solution

$$r = \frac{0.82}{20} = 0.041\,cm$$

and $m = 5.3x10^3$gm

The Stress

$$= \frac{F}{A} = \frac{m\,g}{A} = \frac{(5.3x10^3)980}{\pi\,(0.041)^2} =$$

The strain

$$= \frac{\mathbf{Stress}}{\mathbf{Y}} =$$

$$\mathbf{Strain\ Energy} = \frac{1}{2}(\mathbf{Stress})(\mathbf{Strain}) =$$

Example 2 :

A uniform wire of length 20cm density 0.78gm / cm³ and mass 16gm stretched by a distance 1.2mm when 8kg is supported on it , Find :

The stress in the wire

Young's modulus

The strain energy

Solution

- ..

$$Volume\ V = \frac{m}{\rho} = \frac{16}{7.8} = 2.05\,cm^3$$

$$But\ \ V = A\,.\,L \Rightarrow \quad A = \frac{V}{L} = \frac{2.05}{20} = 0.1\,cm^2$$

$$Stress = \frac{F}{A} = \frac{m\,g}{A} = \frac{(8 \times 10^3)\,980}{0.1} = \qquad dyne / cm^2$$

$$But\ Stress = Y\frac{\Delta L}{L} \quad \Rightarrow \quad Y = \frac{Stress\ .\ 20}{1.2 \times 10^{-1}} = \qquad dyne / cm^2$$

$$Strain\ Energy = \frac{1}{2}(Stress)(Strain) =$$

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Example :

An aluminum cylinder 10cm long with cross-sectional of $20cm^2$ is to be used as a spacer between two steal walls. At 17.2^oC it just slips in between the walls. Then it warms to 22.3^oC. Calculate the stress in the cylinder and the total force it exerted on each wall and the strain energy .

Where ($\alpha = 2.4x10^{-5^o} K^{-1}$, $Y = 7x10^{10}\ N/m^2$)

Solution

Stress

$$= \frac{F}{A} = Y\alpha\Delta T = (7x10^{10})(2.4x10^{-5})(22.3-17.2) = 8.6x10^6\ N/m^2$$

The total force on each wall

$$F = (A)(Stress) = (20x10^{-4})(8.6x10^6) = 1.7x10^4\ N$$

$$Strain\ Energy = \frac{1}{2}(Stress)(Strain)$$

$$But\ Stress = Y(Strain)$$

Example (1): *A 80 Kg mass is hung on a steel wire having 18m long and 3mm diameter. What is the elongation of the wire, knowing Young's modulus for steel is 21×10^{10} N/m²?* كتلة 80 كج علقت فى سلك طوله 18 متر و قطره 3مـم مامقـدار التمدد فى طوله

$m = 80\ kg$ $\qquad$ $L_o = 18\ m$

$2r = 3\ mm = 0.003m$ $\qquad$ $r = 0.0015m$ $\qquad$ $Y = 21 \times 10^{10}\ N/m^2$

Young's modulus is given by $Y = \dfrac{F/A}{\Delta L/L_o}$ so the elongation is $\Delta L = \dfrac{F}{A}\dfrac{L_o}{Y}$

$$\Delta L = \frac{80x9.8}{\pi(0.0015)^2}x\frac{18}{21x10^{10}} = 0.0095m = 9.5mm$$

Example (2)

A piece of copper originally 305 mm long is pulled in tension with a stress of 276 MPa. If the deformation is entirely elastic, what will be the resultant elongation?

$$E = \frac{\sigma}{\varepsilon} \qquad \Rightarrow \qquad \varepsilon = \frac{\sigma}{E}$$

$$\frac{\Delta L}{L_o} = \frac{\sigma}{E} \qquad \Rightarrow \qquad \Delta L = \frac{\sigma L_o}{E}$$

$$\Delta L = \frac{276 * 10^6 * 305 * 10^{-3}}{11 * 10^{10}} = 0.76 * 10^{-3} \ \ \text{m} = 0.76 \ \ \text{mm}$$

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Example (3)

A telephone wire 120 m long and 2.2 mm in diameter is stretched by a force of 380 N. What is the longitudinal stress? If the length after stretching is 120.10 m, what is the longitudinal strain? Determine Young's modulus for the wire?

$$A = \pi r^2 = 3.14 * (1.1*10^{-3})^2 = 3.8 * 10^{-6} \ m^2$$

$$\sigma = \frac{F}{A} = \frac{380}{3.8*10^{-6}} = 100*10^6 \quad N/m^2 = 100 \ MPa$$

$$\Delta L = 120.1 - 120 = 0.1 \quad m$$

$$\varepsilon = \frac{\Delta L}{L_o} = \frac{0.1}{120} = 8.33*10^{-4}$$

$$Y = \frac{\sigma}{\varepsilon} = \frac{100*10^6}{8.33*10^{-4}} = 12*10^{10} \ N/m^2 = 12*10^4 \ MPa$$

Example

In a large lecture hall, a pendulum is to be made by suspending a 40 Kg ball from the end of a steel wire 15 m long.

a) What cross sectional area should the wire have if the applied strength in it is to be only 10% of its breaking strength?

b) How far will the ball stretch the wire ?

(Breaking stress = $0.48*10^9$ N/m^2 , E_s = $200*10^9$ N/m^2)

$$F = m\,g = 40*9.8 = 392 \;\; N$$

$$\sigma = \frac{10}{100}*0.48*10^9 = 48*10^6 \quad N/m^2$$

$$A_o = \frac{F}{\sigma} = \frac{392}{48*10^6} = 8.17*10^{-6} \quad m^2$$

$$Y = \frac{\sigma}{\varepsilon} \qquad \Rightarrow \qquad \varepsilon = \frac{\sigma}{Y}$$

$$\frac{\Delta L}{L_o} = \frac{\sigma}{Y} \qquad \Rightarrow \qquad \Delta L = \frac{\sigma L_o}{Y}$$

$$\Delta L = \frac{48*10^6*15}{200*10^9} = 3.6*10^{-3} \quad m = 3.6 \quad mm$$

EXAMPLE (5)

A load of 102 kg is supported by a wire of length 2 m and cross sectional area 0.1 cm^2. The

$m = 102$ kg $\quad L = 2$ m $\quad A = 0.1\ cm^2 \quad \Delta L = 0.22$ cm

$$\text{Tensile Stress}\,(\sigma) = \frac{F}{A} = \frac{m\,g}{A} = \frac{102*9.8}{0.1*10^{-4}} = 999.6*10^{5}\ \ N/m^{2}$$

$$\text{Tensile Strain}\,(\varepsilon) = \frac{\Delta L}{L_o} = \frac{0.22}{2*100} = 11*10^{-4}$$

$$\text{Young's modulus}\,(Y) = \frac{\sigma}{\varepsilon} = \frac{999.6*10^{5}}{11*10^{-4}} = 90.87*10^{9}\ \ N/m^{2}$$

EXAMPLE (6)

A structure steel rod has a radius R of 9.5 mm and a length L of 81 cm. A force F of $6.2 * 10^4$ N stretches it axially. ($E_{steel} = 2 * 10^{11}$ N/m^2)

(a) What is the stress in the rod ?

(b) What is the elongation of the rod under this load ?

(c) What is the strain?

Radius = 9.5 mm L = 81 cm $F = 6.2 * 10^4$ N

$$\text{Tensile Stress}\,(\sigma) = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{6.2 * 10^4}{\pi (9.5 * 10^{-3})^2} = 2.19 * 10^8 \;\; N/m^2$$

$$\text{Tensile Strain}\,(\varepsilon) = \frac{\Delta L}{L_o} \qquad ; \qquad E = \frac{\sigma}{\varepsilon}$$

$$\Delta L = \varepsilon * L_o = \frac{\sigma}{Y} * L_o = \frac{2.19 * 10^8}{2 * 10^{11}} * 81 * 10^{-2} = 8.86 * 10^{-4} \;\; m$$

$$\text{Tensile Strain}\,(\varepsilon) = \frac{\sigma}{Y} = \frac{2.19 * 10^8}{2 * 10^{11}} = 1.1 * 10^{-3}$$

Example (1): *What fraction of the total volume of an iceberg is exposed? The density of ice is* ρ_{ice} = *0.92 gm/cm³ and that of sea water is* ρ_{water} = *1.03 gm/cm³.*

Solution

إحسب الجزء المعرض للهواء من الثلج الطافى على مياة المحيط

Weight of ice is $W_{ice} = \rho_{ice} V_{ice}\, g$ *the buoyant force of water is* $B = \rho_{water} V_{water}\, g$

$$\rho_{water} V_{water}\, g = \rho_{ice} V_{ice}\, g \quad \Rightarrow \rho_{water} V_{water} = \rho_{ice} V_{ice}$$

$$V_{water}/ V_{ice} = \rho_{ice}/ \rho_{water} = 0.92/1.03 = 89\%$$

The volume of ice exposed in air is 100%-89% = 11%

Example (2): *What is the pressure due to water at a depth of 7.5 Km below sea level? The water density $\rho w = 1.025 \times 10^3\ Kg / m^3$.*

Solution

المطلوب حساب الضغط على عمق 7.5 كم تحت مستوى سطح البحر

$$P = P_o + \rho g h$$

$$= 1.013 \times 10^5 + 1.025 \times 10^3 \times 9.8 \times 7.5 \times 10^3$$

$$= 7.53 \times 10^7\ N / m^2 = 75.3\ MPa$$

Example (3): *What is the pressure at a point 2000m high above sea level assuming that the density of air is approximately constant and $\rho_{air} = 1.22\ Kg / m^3$?*

Solution

المطلوب حساب الضغط على إرتفاع 2000 متر فوق مستوى سطح البحر

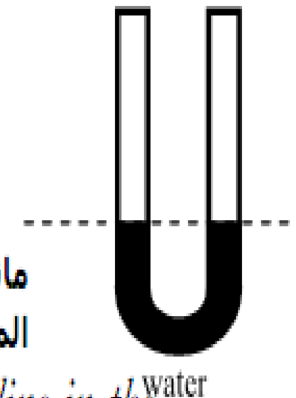
$$P = P_o - \rho_{air}\, g\, h$$

$$= 1.013 \times 10^5 - 1.22 \times 9.8 \times 2000$$

$$= 7.74 \times 10^4\ N / m^2$$

Example (4): *A simple U- tube that is open at both ends is partially filled with water and oil of density 460 kg/m³ is poured into one arm of the tube, forming a column of height* h, *as shown in the Figure. What is the difference in height* h *between the two liquid surfaces?*

Solution



مانومتر به ماء وضع كمية من الزيت فى أحد الفرعين فحدث فرق فى الإرتفاع. إحسب هذا الفرق بين فرعى المانومتر

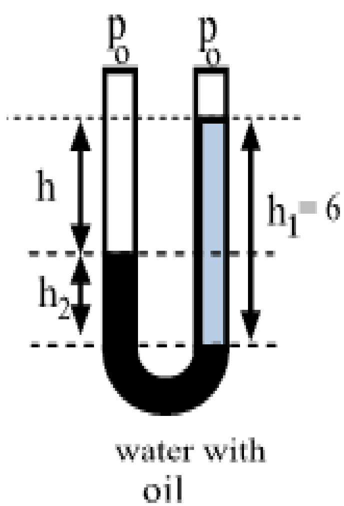
Taking a horizontal line at the low level of the water, the pressure above the horizontal line in the oil branch is $P_{water} = P_o + \rho_{water}\, g\, h_2$

Pressure above the horizontal line in the water branch is $P_{oil} = P_o + \rho_{oil}\, g\, h_1$

Since these two pressures are the same $P_{water} = P_{oil}$

Then, $\cancel{P_o} + \rho_{water}\, g\, h_2 = \cancel{P_o} + \rho_{oil}\, g\, h_1 \quad \Rightarrow \quad \rho_{water}\, h_2 = \rho_{oil}\, h_1$

$1000 \times h_2 = 460 \times 6$ and so $h_2 = 2.76\ cm$, $h = h_1 - h_2 = 3.24\ cm$



Example(6)

A water hose 2.50 cm in diameter is used by a gardener to fill a 30 liter bucket. The gardener notes that it takes 1.00 min to fill the bucket. What is the water speed in the house?

The flow rate = $(30 \times 1000\ cm^3) / (1 \times 60\ s) = 500\ cm^3$

$v = (\text{flow rate} / A = 500\ cm^3) / (\pi\ 1.25^2) = 102\ cm/s = 1.02\ m/s$

Example (7): *A pipe has a diameter of 16 cm at point 1 (P_1 = 200 KPa) and 10 cm at point 2 that is 6 m higher than portion 1. When oil of density 800 kg/ m^3 flows in this pipe at a rate of 0.03 m^3 /s. Find the pressure at point 2?*

Solution

ماسورة يمر بها زيت (كثافته 800 كجم.م$^{-3}$) قطرها 16 سم وضغطها 200 كيلو بسكال عند النقطة 1 و قطرها 10 سم عند النقطة 2 علما بان الزيت يسرى ب 0.03 م^3 لكل ثانية عين الضغط عند النقطة 2؟

$$A_1 v_1 = A_2 v_2 = 0.03 \quad , \textit{then}$$

$v_1 = 0.03 / \pi (0.08)^2 = 1.49 \text{ m/ s}$ $\qquad$ $v_2 = 0.03 / \pi (0.05)^2 = 3.82 \text{ m/s}$

From Bernoulli's Equation

$$P_1 + \tfrac{1}{2} \rho v_1^2 + \rho g y_1 = P_1 + \tfrac{1}{2} \rho v_2^2 + \rho g y_2$$

$$P_2 = P_1 + \tfrac{1}{2} \rho (v_1^2 - v_2^2) + \rho g (y_1 - y_2)$$
$$= 2 \times 10^5 + \tfrac{1}{2}\, 800 \{(1.49)^2 - (3.82)^2\} + 800 \times 9.8 \times 6$$
$$= 1.48 \times 10^5 \text{ Pa.}$$

Example (8): *A Venturi meter reads height h_1 = 30 cm, and h_2 = 10 cm. Find the velocity of flow in the pipe.* $A_1 = 7.85 \times 10^{-3}\ m^2$ and $A_2 = 1.26 \times 10^{-3}\ m^2$.

Solution

$$v_1 = A_2\,[2gh / A_1^2 - A_2^2]^{1/2}$$
$$= 1.26 \times 10^{-3} \times 10^{3}\,[2 \times 20 \times 10^{-2} \times 9.8 / (7.85)^2 - (1.26)^2]^{1/2}$$
$$= 0.32\ m/s$$

Example (1) : A crack in a water tank has a cross-sectional area of $1cm^2$. At what rate is water lost from the tank if the water level in the tank is 4m above the opening?

Solution

Area $A = 1cm^2 = 10^{-4}m^2$ *and height* $h = 4m$

The flow rate at the opening (R)

$$R = A\sqrt{2gh} = (10^{-4}\,m^2)\sqrt{2(9.8m/s^2)(4m)} = 8.85 \times 10^{-4}\,m^3/s$$

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